Antiderivatives:

Antiderivative of a function f is a function F such that

$$F' = f$$

Find antiderivatives of the following functions:

① $f(x) = x^2$

<u>Note</u>: In the power rule, the power gets reduced by 1.

Let, $F(x) = Ax^3$, then $F'(x) = A(x^3)' = 3Ax^2$

$F'(x) = f(x) \Rightarrow \quad 3Ax^2 = x^2$

$\Downarrow$ Equating coefficients we get

$3A = 1$

$\Rightarrow A = \frac{1}{3}$

Hence, $F(x) = \frac{1}{3}x^3$

But $F(x) = \frac{1}{3}x^3 + 100$ also has $f(x)$ as derivative.

So, our general antiderivative is $F(x) = \frac{x^3}{3} + c$, where c is any constant.

⊛ In general the antiderivative of $f(x) = x^n$, $n \neq -1$, is given by

$$F(x) = \frac{x^{n+1}}{n+1} + c, \quad c \text{ is any constant.}$$

⊛ For $n = -1$, $\quad F(x) = \ln x + c$, as $F'(x) = \frac{1}{x} = x^{-1}$.

② $f(x) = 5\cos 3x$

<u>Note</u>:- Since the derivative of trigonometric function is again trigonometric.

Let $F(x) = A\sin 3x + B\cos 3x$

Then, $F'(x) = A\cos 3x \cdot 3 + B(-\sin 3x)\cdot 3$

$$= 3A\cos 3x - 3B\sin 3x$$

Since, $F'(x) = f(x)$

$$\Rightarrow 3A\cos 3x - 3B\sin 3x = 5\cos 3x + 0\sin 3x$$

$$\Rightarrow 3A = 5 \quad \& \quad -3B = 0$$
$$\Rightarrow A = 5/3 \quad \& \quad B = 0$$

Then $F(x) = 5/3\sin 3x + 0\cos 3x$
$= 5/3\sin 3x.$

Hence our general Antiderivative is $F(x) = \frac{5}{3}\sin 3x + c.$

③ $f(x) = e^{mx}$, where m is a fixed constant.

<u>Note</u>:- Since the derivative of an exponential function is again exponential,

Let $F(x) = Ae^{mx}$. Then $F'(x) = Ae^{mx}\cdot m$.

Since $F'(x) = f(x) \Rightarrow Ae^{mx}\cdot m = e^{mx}$

Comparing coefficients we get, $Am = 1 \Rightarrow A = \frac{1}{m}.$

Then $F(x) = \frac{1}{m} e^{mx}$.

So, the general antiderivative is $F(x) = \frac{e^{mx}}{m} + c$.

Ex. Suppose $F'(x) = \frac{1}{\sqrt{x}}$ & $F(1) = 5$. Find $F(x)$.

Here, $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$, i.e., $n = -1/2$. $(\neq -1)$

Then, $F(x) = \frac{x^{-1/2+1}}{-1/2+1} + c$

$$= \frac{x^{1/2}}{1/2} + c$$

$$= 2\sqrt{x} + c$$

Given $F(1) = 5 \Rightarrow 2\sqrt{1} + c = 5$

$\Rightarrow 2 + c = 5 \Rightarrow c = 3$

Hence $F(x) = 2\sqrt{x} + 3$.